

# Holographic Dark Energy Interacting with Two Fluids and Validity of Generalized Second Law of Thermodynamics

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We have considered a cosmological model of holographic dark energy interacting with dark matter and another unknown component of dark energy of the universe. We have assumed two interaction terms  $Q$  and  $Q'$  in order to include the scenario in which the mutual interaction between the two principal components (i.e., holographic dark energy and dark matter) of the universe leads to some loss in other forms of cosmic constituents. Our model is valid for any sign of  $Q$  and  $Q'$ . If  $Q < Q'$ , then part of the dark energy density decays into dark matter and the rest in the other unknown energy density component. But if  $Q > Q'$ , then dark matter energy receives from dark energy and from the unknown component of dark energy. Observation suggests that dark energy decays into dark matter. Here we have presented a general prescription of a cosmological model of dark energy which imposes mutual interaction between holographic dark energy, dark matter and another fluid. We have obtained the equation of state for the holographic dark energy density which is interacting with dark matter and other unknown component of dark energy. Using first law of thermodynamics, we have obtained the entropies for holographic dark energy, dark matter and other component of dark energy, when holographic dark energy interacting with two fluids (i.e., dark matter and other component of dark energy). Also we have found the entropy at the horizon when the radius ( $L$ ) of the event horizon measured on the sphere of the horizon. We have investigated the GSL of thermodynamics at the present time for the universe enveloped by this horizon. Finally, it has been obtained validity of GSL which implies some bounds on deceleration parameter  $q$ .

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## I. INTRODUCTION

Recent observation of the luminosity of type Ia supernovae indicate [1, 2] an accelerated expansion of the universe and the surveys of clusters of galaxies show that the density of matter is very much less than the critical density. This observation leads to a new type of matter which violate the strong energy condition  $\rho + 3p < 0$ . The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as dark energy [3-6]. This mysterious fluid is believed to dominate over the matter content of the Universe by 70 % and to have enough negative pressure as to drive present day acceleration. Most of the dark energy models involve one or more scalar fields with various actions and with or without a scalar field potential [7]. On the other hand when the universe was 380,000 years old neutrinos was 10% atoms i.e. usual baryonic matter was 12%, dark matter was 63%, photons 15% and dark energy was negligible. In the analysis of dark energy the main attraction should be on the state parameter  $w = \frac{p}{\rho}$  where  $p$  and  $\rho$  are the pressure and energy density of the dark energy. In Cosmological constant model  $w = -1$  around present epoch [8] from  $w > -1$  in the near past [9]. There are various kinds of models of dark energy and among all of them, the simplest case is the  $\Lambda$ CDM model which has  $\rho = \Lambda = \text{constant}$  and the equation of state as  $p = -\rho$ . It fits our observational data very well but a problem called Coincidence problem [10] arises in this model, which requires extreme fine-tuning of the order of magnitude  $10^{120}$  of the initial value of  $\Lambda$ .

Now, as the observational data permits us to have a rather time varying equation of state, there are a bunch of models characterized by different scalar fields such as a slowly rolling scalar field (Quintessence) [11], kinetic energy induced K-essence [12], a tachyonic field [13], Chaplygin gas [14], a phantom model [15] and also a quintom model [9]. In a phantom model, we have the equation of state as  $p = \omega\rho$ , where  $\omega < -1$ . The simplest type of phantom model is a scalar field having a potential  $V(\phi)$ , the kinetic energy of which is negative [16]. The quintom model has two scalar fields, one is like that of the quintessence model and the other is like that of the phantom model. The condition  $\omega = -1$  is named as the phantom divide. There are even models which can smoothly cross this phantom divide [17]. The speciality of a phantom model lies in the fact that in these

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type of models the universe ends with a Big Rip singularity [18], which means that in a finite time,  $|p| \rightarrow \infty$ ,  $\rho \rightarrow \infty$  and  $a \rightarrow \infty$ , where  $a(t)$  is the scale factor. Also, the current observation data from Type-Ia supernovae and the CMB anisotropy documents give us limits to the various parameters [19-23] like  $\Omega_B$ ,  $\Omega_{DE}$ ,  $\Omega_{DM}$  where  $\Omega$  denotes the relative density and the suffices  $B$ ,  $DE$ ,  $DM$  represent baryonic matter, dark energy and dark matter respectively. It also gives us data from which we have the limit  $-1.38 < \omega < -0.82$  [24] with a very high level of confidence where  $\omega$  is the equation of state parameter. Recent observations also reveals the fact that our universe is likely to be spatially flat [25].

The holographic principle emerged in the context of black-holes, where it was noted that a local quantum field theory can not fully describe the black holes [26]. Some long standing debates regarding the time evolution of a system, where a black hole forms and then evaporates, played the key role in the development of the holographic principle [27-29]. Cosmological versions of holographic principle have been discussed in various literatures [30-32]. Easther et al [32] proposed that the holographic principle be replaced by the generalized second law of thermodynamics when applied to time-dependent backgrounds and found that the proposition agreed with the cosmological holographic principle proposed by Fischler and Susskind (Ref [30]) for an isotropic open and flat universe with a fixed equation of state. Verlinde [33] studied the holographic principle in the context of an  $(n + 1)$  dimensional radiation dominated closed FRW universe. Numerous cosmological observations have established the accelerated expansion of the universe [34,35]. Since it has been proven that the expansion of the universe is accelerated, the physicists and astronomers started considering the dark energy cosmological observations indicated that at about 2/3 of the total energy of the universe is attributed by dark energy and 1/3 is due to dark matter [36]. In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model [26,36,37]. An approach to the problem of dark energy arises from the holographic principle stated in the first paragraph. For an effective field theory in a box size  $L$  with UV cutoff  $\Lambda_c$ , the entropy  $L^3 \Lambda_c^3$ . The non-extensive scaling postulated by Bekenstein suggested that quantum theory breaks down in large volume [36]. To reconcile this breakdown, Chohen et al [38] pointed out that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. Taking the whole universe into account the largest IR cut-off  $L$  is chosen by saturating the inequality so that we get the holographic dark energy density as [36]  $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$  where  $c$  is a numerical constant and  $M_p \equiv 1/\sqrt{8\pi G}$  is the reduced Plank mass. On the basis of the holographic principle proposed by [30] several others have studied holographic model for dark energy [35]. Employment of Friedman equation [39]  $\rho = 3M_p^2 H^2$  where  $\rho$  is the total energy density and taking  $L = H^{-1}$  one can find  $\rho_m = 3(1 - c^2)M_p^2 H^2$  for flat universe. Thus either  $\rho_m$  or  $\rho_\Lambda$  behaves like  $H^2$ . Thus, dark energy results as pressureless, since  $\rho_\Lambda$  scales like matter energy density  $\rho_m$  with the scale factor  $a$  as  $a^{-3}$ . But, neither dark energy, nor dark matter has laboratory evidence for its existence directly. Also, taking the apparent horizon as the IR cut-off may result in a constant parameter of state  $w$ , which is in contradiction with recent observations implying variable  $w$  [40]. For small value of  $\Omega_k$  in non-flat universe, Setare et al [41] have considered a model as a system which departs slightly from flat space. Consequently, the results for the flat universe they treat the apparent horizon only as an arbitrary distance and not as the systems IR cut-off.

Interaction models where the dark energy weakly interacts with the dark matter have also been studied to explain the evolution of the Universe. This models describe an energy flow between the components. To obtain a suitable evolution of the Universe an interaction is often assumed such that the decay rate should be proportional to the present value of the Hubble parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [42]. These kind of models describe an energy flow between the components so that no components are conserved separately. A variety of interacting holographic dark energy models have been proposed and studied for this purpose [42-45].

Since the discovery of black hole thermodynamics in 1970, physicists have speculated on the thermodynamics of cosmological models in an accelerated expanding universe. In 1973, Bekenstein [46] assumed that there is a relation between the event of horizon and the thermodynamics of a black hole, so that the event of horizon of the black hole is a measure of the entropy of it. This idea has been generalized to horizons of cosmological models, so that each horizon corresponds to an entropy. Thus the second law of thermodynamics was modified in the way that in generalized form, the sum of all time derivative of entropies related to horizons plus time derivative of normal entropy must be positive, i.e. the sum of entropies must be increasing function of time. There is a cosmological event horizon, analogous to a black hole horizon, which can be associated with thermodynamical variables. Supposing that some energy passes through the cosmological event horizon, the definitions of Black Hole temperature and entropy imply that the first law of thermodynamics is valid. In the semiclassical quantum description of black hole physics, it was found that black holes emit Hawking

radiation with a temperature proportional to their surface gravity at the event horizon and they have an entropy which is one quarter of the area of the event horizon in Planck units [47]. The temperature, entropy and mass of black holes satisfy the first law of thermodynamics [48]. On the other hand, it was shown that the Einstein equation can be derived from the first law of thermodynamics by assuming proportionality of the entropy and the horizon area [49]. The Einstein equation for the nonlinear gravitational theory  $f(R)$  was also derived from the first law of thermodynamics with some non-equilibrium corrections [50]. For a general static spherically symmetric spacetime, Padmanabhan [51] showed that the Einstein equation at the horizon gives the first law of thermodynamics on the horizon. The study of the relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context where it was shown that the first law of thermodynamics on the apparent horizon  $\tilde{r}_A$  can be derived from the Friedmann equation and vice versa if we take the Hawking temperature and the entropy on the apparent horizon [52].

The thermodynamics in de Sitter spacetime was first investigated by Gibbons and Hawking in [53]. In a spatially flat de Sitter spacetime, the event horizon and the apparent horizon of the Universe coincide and there is only one cosmological horizon. When the apparent horizon and the event horizon of the Universe are different, it was found that the first law and the second law of thermodynamics hold on the apparent horizon, while they break down if one considers the event horizon [54]. Thermodynamics of the expanding universe has also been the subject of several studies [55-64]. Phantom thermodynamics looks leading to negative entropy of the universe [65] or to appearance of negative temperatures [66]. In accelerated expanding universe, besides the normal entropy, a cosmological horizon entropy can also be considered. One can investigate the conditions for which the generalized second law of thermodynamics (GSL) holds [58, 59]. In these cases GSL asserts that the sum of the horizon entropy, and the normal entropy of the fluid is an increasing function of time. In [58] the change in event-horizon area in cosmological models that depart slightly from de Sitter space was investigated, and it was shown that the area and consequently the (de Sitter) horizon entropy are non decreasing functions of time. In the presence of a viscous fluid, there was found that GSL was satisfied provided that the temperature of the fluid was equal to or lower than de Sitter horizon temperature. Gong et al [67] derived the temperature and entropy of the matter contents inside the apparent horizon from the first law of thermodynamics and discuss the holographic entropy bound and the generalized second law (GSL) of thermodynamics for the Universe with DE. They have addressed the thermodynamics of DE by considering the DE models with constant  $w$  and the generalized Chaplygin gas (GCG).

In the present work, we have considered a cosmological model of holographic dark energy interacting with dark matter and another unknown component of dark energy of the universe. We have assumed two interaction terms  $Q$  and  $Q'$  in order to include the scenario in which the mutual interaction between the two principal components (i.e., holographic dark energy and dark matter) of the universe leads to some loss in other forms of cosmic constituents. In section II, we have presented a general prescription of a cosmological model of dark energy which imposes mutual interaction between holographic dark energy, dark matter and another fluid. We have obtained the equation of state for the holographic dark energy density which is interacting with dark matter and other unknown component of dark energy. In section III, we have obtained the entropies for holographic dark energy, dark matter and other component of dark energy, when holographic dark energy interacting with two fluids (i.e., dark matter and other component of dark energy). We have investigated the validity GSL of thermodynamics at the present time for the universe enveloped by the horizon. Finally, we have presented some concluding remarks in section IV.

## II. HOLOGRAPHIC DARK ENERGY INTERACTING WITH TWO FLUIDS

Assuming the universe to be homogeneous and isotropic, the Friedmann-Robertson-Walker (FRW) metric can be written as

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (1)$$

where  $a(t)$  is the expansion scalar or the scale factor and  $k$  ( $= 0, \pm 1$ ) is the curvature scalar. Then Einstein's field equations become (choosing  $8\pi G = c = 1$ )

$$3H^2 + \frac{3k}{a^2} = \rho_\Lambda + \rho_m + \rho_X \quad (2)$$

and

$$2\dot{H} - \frac{2k}{a^2} = -[(\rho_\Lambda + \rho_m + \rho_X) + (p_\Lambda + p_m + p_X)] \quad (3)$$

where  $\rho_\Lambda$ ,  $\rho_m$ ,  $\rho_X$  and  $p_\Lambda$ ,  $p_m$ ,  $p_X$  are respectively energy density and pressure of holographic dark energy, dark matter and another unknown component of dark energy. We will assume that the dark matter component is interacting with the holographic dark energy component, so their continuity equations take the form [68]

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q' \quad (4)$$

and

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q \quad (5)$$

where  $Q$  and  $Q'$  in order to include the scenario in which the mutual interaction between the two principal components of the universe leads to some loss in other forms of cosmic constituents. In this case, we have assumed  $Q \neq Q'$ , so the continuity equation for other component of dark energy becomes

$$\dot{\rho}_X + 3H(\rho_X + p_X) = Q' - Q \quad (6)$$

If  $Q < Q'$ , then part of the dark energy density decays into dark matter and the rest in the other unknown energy density component. But if  $Q > Q'$ , then dark matter energy receives from dark energy and from the unknown component of dark energy.

We are taking about in this case that dark energy decay into dark matter (or vice versa, depending on the sign of  $Q$ ) and other component. Assume, the interaction terms  $Q$  and  $Q'$  are [69]

$$Q = \Gamma_m \rho_\Lambda, \quad Q' = \Gamma_\Lambda \rho_\Lambda \quad (7)$$

where,  $\Gamma_\Lambda$  is the decaying rate of energy from holographic dark energy to dark matter and other unknown component of dark energy and  $\Gamma_m$  is the receiving rate of energy from holographic dark energy to dark matter only.

Consider the equation of state:

$$p_\Lambda = w_\Lambda \rho_\Lambda, \quad p_m = w_m \rho_m, \quad p_X = w_X \rho_X \quad (8)$$

and assume the ratios for energy densities:

$$r_1 = \frac{\rho_m}{\rho_\Lambda}, \quad r_2 = \frac{\rho_X}{\rho_\Lambda} \quad (9)$$

So from the above continuity equations, we obtain

$$\dot{r}_1 = r_1 \Gamma_\Lambda + \Gamma_m + 3H(w_\Lambda - w_m)r_1 \quad (10)$$

and

$$\dot{r}_2 = (1 + r_2)\Gamma_\Lambda - \Gamma_m + 3H(w_\Lambda - w_X)r_2 \quad (11)$$

Define:

$$w_m^{eff} = w_m - \frac{\Gamma_m}{3r_1 H}, \quad w_\Lambda^{eff} = w_\Lambda + \frac{\Gamma_\Lambda}{3H}, \quad w_X^{eff} = w_X + \frac{\Gamma_m - \Gamma_\Lambda}{3r_2 H} \quad (12)$$

so that the continuity equations (4) - (6) become

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{eff})\rho_\Lambda = 0 \quad (13)$$

$$\dot{\rho}_m + 3H(1 + w_m^{eff})\rho_m = 0 \quad (14)$$

and

$$\dot{\rho}_X + 3H(1 + w_X^{eff})\rho_X = 0 \quad (15)$$

Now define the density parameters:

$$\Omega_m = \frac{\rho_m}{3H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2}, \quad \Omega_X = \frac{\rho_X}{3H^2}, \quad \Omega_k = \frac{k}{a^2 H^2} \quad (16)$$

so from the field equation (2), we obtain

$$\Omega_m + \Omega_\Lambda + \Omega_X = 1 + \Omega_k \quad (17)$$

which implies

$$\dot{\Omega}_m + \dot{\Omega}_X = \dot{\Omega}_k - \dot{\Omega}_\Lambda \quad (18)$$

From equations (9) and (17), we have

$$r_1 = \frac{\Omega_m}{\Omega_\Lambda} = \frac{1 + \Omega_k - \Omega_\Lambda - \Omega_X}{\Omega_\Lambda} \quad (19)$$

and

$$r_2 = \frac{\Omega_X}{\Omega_\Lambda} = \frac{1 + \Omega_k - \Omega_\Lambda - \Omega_m}{\Omega_\Lambda} \quad (20)$$

Now for non-flat universe, the energy density for holographic dark energy is

$$\rho_\Lambda = 3c^2 L^{-2} \quad (21)$$

where  $c (\geq 1)$  is a constant and  $L$  represents the radius of the event horizon measured on the sphere of the horizon defined by

$$L = ar(t) \quad (22)$$

where  $r(t)$  is a future event horizon obtained from the following equation

$$r(t) = \frac{\sin y}{\sqrt{k}} \quad (23)$$

where  $y = \frac{\sqrt{k}R_h}{a}$ ,  $R_h$  is the radial size of the event horizon which is measured in  $r$  direction defined by

$$R_h = a \int_t^\infty \frac{dt}{a} \quad (24)$$

Now from definition of  $\Omega_\Lambda$  and using (21), we obtain

$$L = \frac{c}{H\sqrt{\Omega_\Lambda}} \quad (25)$$

From (21) - (25), we have

$$\dot{L} = \frac{c}{\sqrt{\Omega_\Lambda}} - \cos y \quad (26)$$

From (16), (21), (22) and (23), we have

$$\cos y = \sqrt{1 - c^2 \frac{\Omega_k}{\Omega_\Lambda}} \quad (27)$$

Using (12), (14), (21) and (27), we get the equation of state for holographic dark energy as

$$w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} - \frac{\Gamma_\Lambda}{3H} \quad (28)$$

### III. GENERALIZED SECOND LAW OF THERMODYNAMICS

We consider the FRW universe as a thermodynamical system with the horizon surface as a boundary of the system. To study the generalized second law (GSL) of thermodynamics through the universe we deduce the expression for normal entropy using the first law of thermodynamics i.e.,  $TdS = PdV + dE$ , where,  $T$ ,  $S$ ,  $P$ ,  $V$  and  $E$  are respectively temperature, entropy, pressure, volume and internal energy within the event horizon (of radius  $L$  which is measured on the sphere of the horizon) of the universe. The entropies for holographic dark energy, dark matter and other component of dark energy are given by [69]

$$dS_\Lambda = \frac{1}{T}(P_\Lambda dV + dE_\Lambda) \quad (29)$$

$$dS_m = \frac{1}{T}(P_m dV + dE_m) \quad (30)$$

and

$$dS_X = \frac{1}{T}(P_X dV + dE_X) \quad (31)$$

where  $V = \frac{4\pi L^3}{3}$  is the volume containing matter and dark energies with

$$E_\Lambda = \frac{4\pi L^3 \rho_\Lambda}{3}, \quad P_\Lambda = w_\Lambda^{eff} \rho_\Lambda \quad (32)$$

$$E_m = \frac{4\pi L^3 \rho_m}{3}, \quad P_m = w_m^{eff} \rho_m \quad (33)$$

and

$$E_X = \frac{4\pi L^3 \rho_X}{3}, \quad P_X = w_X^{eff} \rho_X \quad (34)$$

Assuming,  $T = \frac{1}{2\pi L}$  and  $x = \log a$  and using equations (12), (25), (26), (29) and (32), we obtain

$$\frac{dS_\Lambda}{dx} = \frac{24\pi^2 c^2 L \dot{L}}{H} \left( w_\Lambda + \frac{\Gamma_\Lambda}{3H} + \frac{1}{3} \right) \quad (35)$$

$$\frac{dS_m}{dx} = 8\pi^2 L \left[ \left( 3w_m H - \frac{\Gamma_m}{r_1} \right) \Omega_m L^2 \dot{L} + c^2 \left( \frac{\Omega_m \dot{L}}{\Omega_\Lambda H} + \frac{L \dot{\Omega}_m}{\Omega_\Lambda H} - \frac{L \Omega_m}{H \Omega_\Lambda^2} \dot{\Omega}_\Lambda \right) \right] \quad (36)$$

and

$$\frac{dS_X}{dx} = 8\pi^2 L \left[ \left( 3w_X H + \frac{\Gamma_m - \Gamma_\Lambda}{r_2} \right) \Omega_X L^2 \dot{L} + c^2 \left( \frac{\Omega_X \dot{L}}{\Omega_\Lambda H} + \frac{L \dot{\Omega}_X}{\Omega_\Lambda H} - \frac{L \Omega_X}{H \Omega_\Lambda^2} \dot{\Omega}_\Lambda \right) \right] \quad (37)$$

Now entropy at the horizon is given by

$$S_L = \pi L^2 \quad (38)$$

so that from equations (25) and (26), we obtain

$$\frac{dS_L}{dx} = \frac{2\pi c}{H^2 \sqrt{\Omega_\Lambda}} \left( \frac{c}{\sqrt{\Omega_\Lambda}} - \cos y \right) \quad (39)$$

From equations (14), (15), (16) and (21), we have

$$3w_m H - \frac{\Gamma_m}{r_1} = -H + \frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda} - \frac{\dot{\Omega}_m}{\Omega_m} - \frac{2H}{c} \sqrt{\Omega_\Lambda} \cos y \quad (40)$$

and

$$3w_X H + \frac{\Gamma_m - \Gamma_\Lambda}{r_2} = -H + \frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda} - \frac{\dot{\Omega}_X}{\Omega_X} - \frac{2H}{c} \sqrt{\Omega_\Lambda} \cos y \quad (41)$$

Using equations (16) and (21) and defining the deceleration parameter  $q = -1 - \frac{\dot{H}}{H^2}$  we can obtain

$$\dot{\Omega}_k = 2q H \Omega_k \quad (42)$$

and

$$\dot{\Omega}_\Lambda = \frac{2\Omega_\Lambda}{\Omega_k} \left( H \Omega_\Lambda - L^{-1} \dot{L} \Omega_\Lambda + q H \Omega_k \right) \quad (43)$$

Using (18), (28), (35)-(41) we get,

$$\begin{aligned} \frac{d}{dx}(S_\Lambda + S_m + S_X + S_L) &= \frac{2\pi L \dot{L}}{H} + 8\pi^2 L^3 \dot{L} \left[ \left( -H + \frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda} - \frac{2H}{c} \sqrt{\Omega_\Lambda} \cos y \right) (\Omega_m + \Omega_X) + (\dot{\Omega}_\Lambda - \dot{\Omega}_k) \right] \\ &+ 8\pi^2 c^2 L \left[ \frac{(\Omega_m + \Omega_X) \dot{L}}{\Omega_\Lambda H} + \frac{L(\dot{\Omega}_k - \dot{\Omega}_\Lambda)}{\Omega_\Lambda H} - \frac{L(\Omega_m + \Omega_X)}{H \Omega_\Lambda^2} \dot{\Omega}_\Lambda \right] - \frac{16\pi^2 c L \dot{L} \sqrt{\Omega_\Lambda} \cos y}{H} \end{aligned} \quad (44)$$

Now putting the values of  $L$ ,  $\dot{L}$ ,  $\cos y$ ,  $\Omega_X$ ,  $\dot{\Omega}_k$  and  $\dot{\Omega}_\Lambda$  from equations (17), (20)-(22), (42) and (43), we finally get

$$\begin{aligned} \frac{d}{dx}(S_\Lambda + S_m + S_X + S_L) = & \frac{2\pi c}{H^2 \Omega_k \Omega_\Lambda^2} [-8\pi c(1 + \Omega_k)(\Omega_\Lambda^2 + c^2 \Omega_k^2) + c\Omega_k \Omega_\Lambda \{1 + 8\pi(1 + c^2)(1 + \Omega_k)\} \\ & - \Omega_k \sqrt{\Omega_\Lambda - c^2 \Omega_k} \{\Omega_\Lambda + 8\pi c^2(1 + q + \Omega_k)\}] \end{aligned} \quad (45)$$

We have seen that r.h.s. of the expression (44) depends on  $c$ ,  $H$ ,  $q$ ,  $\Omega_k$  and  $\Omega_\Lambda$ . At the present time, setting  $c = 1$ ,  $\Omega_k = 0.01$  and  $\Omega_\Lambda = 0.72$ , we obtain

$$\frac{d}{dx}(S_\Lambda + S_m + S_X + S_L) = -\frac{15767 + 257q}{H^2} \quad (46)$$

From the above expression we see that  $\frac{d}{dx}(S_\Lambda + S_m + S_X + S_L) \geq 0$  if  $q \leq -61.43$ . But at the present time,  $q > -1$ , so GSL can not be satisfied in our model.

#### IV. DISCUSSIONS

In this work, we have considered FRW model of the universe filled with 3 fluids i.e., holographic dark energy, dark matter and another unknown component of dark energy. We have considered a cosmological model of holographic dark energy interacting with dark matter and another unknown component of dark energy of the universe. We have assumed two interaction terms  $Q$  and  $Q'$  in order to include the scenario in which the mutual interaction between the two principal components (i.e., holographic dark energy and dark matter) of the universe leads to some loss in other forms of cosmic constituents. Our model is valid for any sign of  $Q$  and  $Q'$ . If  $Q < Q'$ , then part of the dark energy density decays into dark matter and the rest in the other unknown energy density component. But if  $Q > Q'$ , then dark matter energy receives from dark energy and from the unknown component of dark energy. Observation suggests that dark energy decays into dark matter. We have presented a general prescription of a cosmological model of dark energy which imposes mutual interaction between holographic dark energy, dark matter and another fluid. We have obtained the equation of state for the holographic dark energy density which is interacting with dark matter and other unknown component of dark energy. Using first law of thermodynamics, we have obtained the entropies for holographic dark energy, dark matter and other component of dark energy, when holographic dark energy interacting with two fluids (i.e., dark matter and other component of dark energy). Also we have found the entropy at the horizon when the radius ( $L$ ) of the event horizon measured on the sphere of the horizon. We have investigated the GSL of thermodynamics at the present time for the universe enveloped by this horizon. Finally, it has been obtained validity of GSL which implies some bounds on deceleration parameter  $q$ . But at the present time,  $q > -1$ , so GSL can not be satisfied in our model.

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